RY-003-1016002

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Seat No. ________

B. Sc. (Sem. VI) (CBCS) Examination

March - 2019

Mathematics : Paper - M-09(A)

(Mathematical Analysis-2 & Group Theory-2)

Faculty Code : 003
Subject Code : 1016002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Write answer of each question in your main answer sheet.

1 (A) Answer the following in brief : **4**

(1) Define Totally bounded set.

(2) Define : Connected set & Disconnected set.

(3) Determine whether the subset {2,–3} of metric space R is compact or not.

(4) Define compact metric space.

(B) Attempt any **one** out of **two** : **2**

(1) Show that subset R-{7} is not connected.

(2) Show that every finite subset of a metric space is compact.

(C) Attempt any **one** out of **two** : **3**

(1) State and prove Bolzano-Weirstrass theorem.

(2) If F is a closed subset of metric pace X and K is a compact subset of X, then prove that $F \cap K$ is also compact.

(D) Attempt any **one** out of **two** : **5**

(1) State and prove theorem of nested intervals.

(2) Prove that continuous image of connected set is connected.

2 (A) Answer the following questions in brief : **4**

(1) Define Laplace Transform.

(2) Find $L^{-1}\left(\frac{1}{s-3}\right)$.

(3) Find $L^{-1}\left(\frac{1}{s^2+4}\right)$

(4) Show that $L(1)=\frac{1}{s}$, where $s>0$.

(B) Attempt any **one** out of **two** : **2**

(1) Find $L^{-1}\left(\frac{s+2}{(s-2)^3}\right)$.

(2) Find $L(2t+5\sin 3t)$

(C) Attempt any **one** out of **two** : **3**

(1) Find Laplace transform of $\sqrt{te}^{2t}$.

(2) If $L\{f(t)\}=\overline{f}(s)$ then prove that

$L\{e^{at}f(t)\}=\overline{f}(s-a)\cdot$

(D) Attempt any **one** out of **two** : **5**

(1) If $f(t)=e^t, t\le 2$

$= 3, t>2$ then find $L\{f(t)\}$.

(2) Prove that $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)=\frac{1}{2a}t\sin at$.

3 (A) Answer the following quesitons in briefly : **4**

(1) Find $L(t^2e^{at})$

(2) Write convolution theorem.

(3) Find $L(t\sin at)$.

(4) Find $L\left(\frac{\sin t}{t}\right)$.

(B) Attempt any **one** out of **two** : **2**

(1) If $L\{f(t) = \overline{f}(s)$ then prove

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}[\overline{f}(s)].$$

(2) If $L\{f(t)\} = \overline{f}(s)$ then prove

$$L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f}(s)ds.$$

(C) Attempt any **one** out of **two** : **3**

(1) Prove that $L\left\{\dfrac{e^{-at} - e^{-bt}}{t}\right\} = \log\left(\dfrac{s+b}{s+a}\right)$.

(2) Prove that $L^{-1}\left(\log\left(\dfrac{s+b}{s+a}\right)\right) = \dfrac{e^{-at} - e^{-bt}}{t}$.

(D) Attempt any **one** out of **two** : **5**

(1) Prove that $L^{-1}\left\{\dfrac{s^2 - a^2}{(s^2 + a^2)^2}\right\} = t\cos at$.

(2) Using convolution theorem, prove

$$L^{-1}\left\{\frac{1}{s(s^2+4)}\right\} = \frac{1}{4}(1 - \cos 2t).$$

4 (A) Answer the following questions in briefly : **4**

(1) Define Epimorphism.

(2) Define homomorphism.

(3) If $\phi : (G, *) \to (G', \Delta), \phi(x) = e', \forall \times \in G$ is a homomorphism. Then find $K_\varnothing$.

(4) Define Kernel of homomorphism.

(B) Attempt any **one** out of **two** : **2**

(1) Let $\phi : (G, *) \to (G', \Delta)$ is homomorphism. If $H' \leq G'$ then prove $\phi^{-1}(H) \leq G$.

(2) If $\phi : (G, *) \to (G', \Delta)$ is homomorphism. Then $\phi(e) = e'$ where e & e' are identity elements of G &G' respectively.

(C) Attempt any **one** out of **two** : **3**

(1) Find all homomorphism's of $(Z,+)$ onto $(Z,+)$.

(2) Prove that A Homomorphism $\phi:(G,*)\to(G',\Delta)$ is one-one iff $k_\phi=\{e\}$.

(D) Attempt any **one** out of **two** : **5**

(1) State and prove first fundamental theorem of homomorphism.

(2) If $\phi:(G,*)\to(G',\Delta)$ is a Homomorphism. Then prove that Kernel K_ϕ is a normal Subgroup of G.

5 (A) Answer the following questions in briefly : **4**

(1) Define Subring.

(2) If polynomial f = (5,0,0,0,0,.......) then find order of f.

(3) Give an example of a ring without unity.

(4) Define Monic polynomial.

(B) Attemtp any **one** out of **two** : **2**

(1) Find inverse of quaternion 1 + i + j + k.

(2) If $f(x)=(2,3,4,2,0,0.....)$ and $g(x)=(4,2,0,0,3,0.....)\in R[x]$ then find $f(x)+g(x)$.

(C) Attempt any **one** out of **two** : **3**

(1) State and prove Remainder theorem of polynomials.

(2) In $R[x], f(x)=4x^4-3x^2+1$ is divided by $g(x)=x^3-2x+1$ then find quotient $q(x)$ and remainder $r(x)$.

(D) Attempt any **one** out of **two** : **5**

(1) State and prove division algorithm for polynomials.

(2) State and prove factor theorem of polynomials.
